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The Lindelöf principle in several complex variables

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Abstract

We formulate and prove a new Lindelöf principle in the function theory of several complex variables. Inspired by the classical result, as improved later by Lehto and Virtanen, this new result meshes closely with the well-established Fatou theorems of Koranyi and Stein. In particular, this is a Lindelöf principle for *admissible* approach regions. We further adapt the new principle to the Levi geometry of various domains. The results in this paper improve on earlier results of Cirka, Cima/Krantz, Abate, and Abate/Tauraso.

Keywords: Lindelöf principle; Boundary behavior; Admissible approach

0. Introduction

The most classical formulation of the Lindelöf principle on the disc (see [16]) is as follows:

Theorem. *Let f be a bounded, holomorphic function on the unit disc $D \subseteq \mathbb{C}$. Suppose that the radial limit*

$$\lim_{r\to 1^-} f(re^{i\theta}) \equiv \lambda \in \mathbb{C} \tag{$*$}$$

of f exists at the boundary point $e^{i\theta}$. Then in fact f has nontangential limit λ at $e^{i\theta}$.

Thus we have a sort of tauberian theorem: for *bounded* holomorphic functions, radial convergence implies nontangential convergence. It is of interest to have a result of this nature in several

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complex variables. Pioneering work on the Lindelöf principle in several complex variables was done by Cirka [4].

The paper [3] established a Lindelöf principle for holomorphic functions of several variables. That result was new and optimal in the following senses:

(i) It was proved for *normal functions* (see [14]), in a sense the most natural function space for which to consider a Lindelöf principle.
(ii) It was formulated in terms of the Kobayashi metric, thus providing an "optimal" result in terms of the intrinsic Levi geometry of the domain.

But the result of [3] has certain drawbacks. Notable among these is that, whereas the most natural mode of boundary convergence in the several variable setting is admissible convergence (see [10–12,19]), that used in the results of [3] is hypoadmissible converge—a strictly weaker concept. The result formulated in [3] is in fact *false* for admissible convergence (see below). In addition, when the radial curve, as in $(*)$, is replaced by a fairly arbitrary curve—especially by a curve with a significant complex tangential component—then a rather unsatisfying result obtains.

In the present paper we introduce new techniques that address the shortcomings of [3] and produce a sharp version of the Lindelöf principle. In common with [3], we shall be able to prove our result not only for bounded holomorphic functions but also for normal functions. We refer the reader also to [1,2] for modern work on the Lindelöf principle in several variables.

1. Definitions and basic concepts

It is perhaps well to review some key ideas about the boundary limits of holomorphic functions of several complex variables. More detailed background may be found in [12] (see also [15]).

First consider the domain which is the unit ball B in $\mathbb{C}^n$. If $P = (p_1, p_2, \ldots, p_n) \in \partial B$ and $\alpha > 1$, then we set the admissible approach region of aperture α at P be

$$\mathcal{A}_\alpha(P) = \bigl\{z \in B\colon |1 - z \cdot \bar{P}| < \alpha\bigl(1 - |z|\bigr)\bigr\}.$$

Here $z \cdot \bar{P} \equiv \sum_j z_j \bar{P}_j$. This region is of nontangential shape in the complex normal direction at P and of parabolic shape in the complementary, complex tangential direction.

Now let $\Omega \subseteq \mathbb{C}^n$ be a smoothly bounded domain (in fact C^2 boundary will usually suffice). Let $P \in \partial\Omega$ and $\alpha > 1$. We let $T_P(\partial\Omega)$ denote the ordinary real tangent space (of real dimension $(2n-1)$) to $\partial\Omega$ at P. Also $\mathcal{T}_P(\partial\Omega)$ denotes the complex tangent space (of complex dimension $(n-1)$) at P (again see [12] for details). If $z \in \Omega$ is near P then we let

$$\delta_P(z) = \min\bigl\{\mathrm{dist}_{\mathrm{Eucl}}(z, \partial\Omega), \mathrm{dist}_{\mathrm{Eucl}}\bigl(z, T_P(\partial\Omega)\bigr)\bigr\}.$$

Notice, that if Ω is convex, then $\delta_P(z) = \delta_\Omega(z) \equiv \mathrm{dist}_{\mathrm{Eucl}}(z, \partial\Omega)$. If $P \in \partial\Omega$, let ν_P denote the unit outward normal vector to $\partial\Omega$ at P. If $\alpha > 1$, let the (classical) admissible approach region of aperture α at P be

$$\mathcal{A}_\alpha = \bigl\{z \in \Omega\colon \bigl|(z - P) \cdot \bar{\nu}_P\bigr| < \alpha\delta_P(z),\ |z - P|^2 < \alpha\delta_P(z)\bigr\}.$$

These are the definitions of Stein [19]. They are optimal (in several measurable respects to be indicated below) on the ball B and on strongly pseudoconvex domains.

When we do harmonic analysis on a domain Ω in $\mathbb{C}^n$, we equip the boundary with $(2n-1)$-dimensional Hausdorff measure. We let $H^p(\Omega)$ denote the usual Hardy space (see [12, Chapter 8]), calculated with respect to this measure. That is, if ρ is a defining function for Ω and $\epsilon > 0$ is small, we let

$$\partial\Omega_\epsilon = \bigl\{z \in \Omega\colon\ \rho(z) = -\epsilon\bigr\}.$$

Let $d\sigma_\epsilon$ denote Hausdorff $(2n-1)$-dimensional measure on $\partial\Omega_\epsilon$. Then define, for $0 < p < \infty$,

$$H^p(\Omega) = \left\{ f \text{ holomorphic on } \Omega\colon\ \sup_{\epsilon>0} \int\limits_{\partial\Omega_\epsilon} \bigl|f(z)\bigr|^p \, d\sigma_\epsilon(z) < \infty \right\}.$$

Of course $H^\infty(\Omega)$ is the bounded, holomorphic functions on Ω equipped with the usual norm.

Now Koranyi's theorem [10,11] is formulated as follows. Let $H^p(B)$ be the usual Hardy space on the ball as discussed in [12]. We let $0 < p \leqslant \infty$ as usual.

Theorem 1. *Let f be an H^p function on the unit ball $B \subseteq \mathbb{C}^n$. Then, for almost every $P \in \partial B$,*

$$\lim_{\mathcal{A}_\alpha(P) \ni z \to P} f(z) \equiv \tilde{f}(P)$$

exists.

In 1972 (see [19]), Stein generalized this result as follows:

Theorem 2. *Let f be an H^p function on the domain $\Omega \subseteq \mathbb{C}^n$ with C^2 boundary. Then, for almost every $P \in \partial\Omega$,*

$$\lim_{\mathcal{A}_\alpha(P) \ni z \to P} f(z) \equiv \tilde{f}(P)$$

exists.

In subsequent years, the papers [13,17,18] (among others) adapted these results to the Levi geometry of the domain Ω, so that the approach regions were broader than quadratic at points of type greater than 2 (in the sense of Catlin/D'Angelo/Kohn). In view of all this development, it would be natural to expect that the proper Lindelöf principle on a domain in several complex variables would be formulated in terms of admissible convergence adapted to the Levi geometry. Unfortunately, as the next example illustrates, such an expectation fails.

Example 3. Let B be the unit ball in $\mathbb{C}^2$. Define

$$f(z_1, z_2) = \frac{z_2^2}{1 - z_1}.$$

The simple inequalities

$$|z_2|^2 < 1 - |z_1|^2 = \bigl(1 - |z_1|\bigr)\bigl(1 + |z_1|\bigr) < 2\bigl(1 - |z_1|\bigr) \leqslant 2|1 - z_1|$$

show that f is bounded.

Let $P = (1, 0) \equiv \mathbb{1} \in \partial B$. Notice that

$$\lim_{r \to 1^-} f(rP) = \lim_{r \to 1^-} 0 = 0.$$

But if $p^j = (1 - 1/j, 1/\sqrt{j})$ for $j = 4, 5, \ldots$, then

$$f(p^j) = \frac{1/j}{1/j} = 1.$$

Of course the points p^j all lie in the admissible approach region $\mathcal{A}_2(P)$. So we see that the admissible limit of f at P is 1 while the radial limit is 0.

See [12, Section 8.7] for further discussion of this example and its ramifications.

The traditional wisdom, in view of this last example, has been that the expected version of the Lindelöf principle—formulated in terms of admissible convergence—fails. One of the main purposes of the present paper is to correct this situation, indeed to present a version of the Lindelöf principle that *is* valid for admissible convergence. In the next section we present a preliminary version of the result just to give a sense of what we are about.

2. A preliminary version of the main result

We begin by proving the following proposition and discussing it.

Proposition 4. *Let f be a bounded, holomorphic function on the unit ball $B \subseteq \mathbb{C}^2$. Let*

$$T = \left\{(s + i0, t + i0) \in \mathbb{C}^2\colon\ s, t \in \mathbb{R},\ 0 < s < 1,\ 0 < |t| < \sqrt{2 - 2s}\right\}.$$

Let $\mathbb{1} = (1 + i0, 0 + i0)$. Suppose that

$$\lim_{T \ni z \to \mathbb{1}} f(z) \equiv \lambda \in \mathbb{C}$$

exists. Then, for any $\alpha > 1$,

$$\lim_{\mathcal{A}_\alpha(\mathbb{1}) \ni z \to \mathbb{1}} f(z) = \lambda.$$

Proof. The proof follows classical lines. We may assume that $\lambda = 0$. Define, for $j = 1, 2, \dots,$

$$\Omega_j = \Big\{(z_1, z_2) \in \mathbb{C}^2\colon\ 1 - 2^{-j+1}/(2\alpha) \leqslant \operatorname{Re} z_1 < 1 - 2^{-j-1}/(2\alpha),\\ |\operatorname{Im} z_1| < 2^{-j+1},\ |z_2| < \sqrt{2^{-j+1}}/\sqrt{\alpha}\Big\}.$$

Notice that

$$\bigcup_{j=1}^{\infty} \Omega_j \supseteq \mathcal{A}_\alpha(\mathbb{1}).$$

Observe that, for $1 \leqslant j_0 \in \mathbb{Z}$, the map

$$\varphi_j(z_1, z_2) = \left(2^{j-j_0}(z_1 - 1) + 1, \sqrt{2^{j-j_0}}z_2\right)$$

sends Ω_j biholomorphically onto Ω_{j_0}. Also, following the paradigm of the classical Lindelöf principle on the disc, we may see that

$$\bigcup_{j=1}^{\infty} \Omega_j = \bigcup_{j=1}^{\infty} \varphi_j^{-1}(\Omega_{j_0}) \supseteq \mathcal{A}_\alpha(\mathbb{1}).$$

Furthermore, we see that

$$T \cap \Omega_j = \left\{(s, t)\colon\ 1 - 2^{-j+1}/(2\alpha) \leqslant s < 1 - 2^{-j-1}/(2\alpha),\ 0 < t < \sqrt{2 - 2s}\right\}.$$

Of course φ_j maps $T \cap \Omega_j$ onto

$$T \cap \Omega_{j_0} = \left\{(s, t)\colon\ 1 - 2^{-j_0+1}/(2\alpha) \leqslant s < 1 - 2^{-j_0-1}/(2\alpha),\ 0 < t < \sqrt{2 - 2s}\right\}.$$

Now consider the function

$$g_j \equiv f \circ \varphi_j^{-1} : \Omega_{j_0} \to \mathbb{C}.$$

Being uniformly bounded, the g_j form a normal family. Let g_0 be a subsequential limit function.

It follows from our hypotheses that g_0 vanishes on $T \cap \Omega_{j_0}$. But $T \cap \Omega_{j_0}$, being two-dimensional and totally real, is a set of determinacy for holomorphic functions. Hence $g_0 \equiv 0$.

Unraveling the logic, we find that if $K \subseteq \Omega_{j_0}$ is a compact set such that

$$\bigcup_{j=1}^{\infty} \varphi_j^{-1}(K) \supseteq \mathcal{A}_\alpha(\mathbb{1}),$$

then $g_j \to 0$ uniformly on K. It follows that f itself has admissible limit 0 on K. $\square$

The proof that we have just presented is misleadingly simple. For it is an artifact of the special geometry of T, and the way that it is imbedded in space, that $\varphi_j^{-1}(T \cap \Omega_j) = T \cap \Omega_{j_0}$ for each j. For a very general sort of Lindelöf principle, we would like to replace the flat T with a rather arbitrary, two-dimensional, totally real surface. In that situation, the sets

$$\varphi_j^{-1}(T \cap \Omega_j)$$

could be pairwise disjoint. Thus additional arguments will be required. It is worth noting that this problem arises even on the disc in the complex plane—in the situation where the hypothesis of the Lindelöf principle is the existence of a limit along a somewhat arbitrary curve rather than along a radius.

With these thoughts in mind, we now formulate a more sophisticated version of the Lindelöf principle on the ball:

Proposition 5. *Let f be a bounded, holomorphic function on the unit ball $B \subseteq \mathbb{C}^2$. Let*

$$T = \bigl\{(s + i0, t + i0):\ s, t \in \mathbb{R},\ 0 < s < 1,\ 0 < |t| < \sqrt{2 - 2s}\bigr\}.$$

Suppose that $\rho : T \to \mathbb{R}^2$ is a C^2 function, with bounded first and second derivatives, such that (writing $\rho(s,t) = (\rho_1(s,t), \rho_2(s,t))$)

$$\mathcal{T} = \bigl\{\bigl(s + i\rho_1(s,t), t + i\rho_2(s,t)\bigr):\ (s,t) \in T\bigr\}$$

is a two-dimensional, totally real manifold in $B \subseteq \mathbb{C}^2$.

Let $\mathbb{1} = (1 + i0, 0 + i0)$. Suppose that

$$\lim_{\mathcal{T} \ni z \to \mathbb{1}} f(z) \equiv \lambda \in \mathbb{C}$$

exists. Then, for any $\alpha > 1$,

$$\lim_{\mathcal{A}_\alpha(\mathbb{1}) \ni z \to \mathbb{1}} f(z) = \lambda.$$

This result is more nearly like the general Lindelöf principle that one may find in [14]. It hypothesizes the existence of a limit along a fairly "arbitrary" surface terminating at the boundary point $\mathbb{1} \in \partial B$, and it concludes the existence of an admissible limit. It still leaves open the question of obtaining a result on more general domains, and also the question of treating normal functions. We save those two topics for subsequent sections.

Proof. Let Ω_j, ϕ_j, g_j be as in the proof of the preceding proposition. As before, we may assume that $\lambda = 0$ and we may obtain a subsequential limit function g_0.

But it is important for us now to note that the sets

$$\mathcal{T}_j \cap \Omega_{j_0} \equiv \varphi_j(\mathcal{T} \cap \Omega_j)$$

are all graphs over $T \cap \Omega_{j_0}$ of functions τ_j, and each τ_j has, by design, bounded derivatives (with the bound uniform in j). Thus we may extract (using the Ascoli–Arzela theorem) a subsequence τ_{j_k} that converges uniformly, along with its first derivatives, on compacta to some τ_0. We pass to a corresponding subsequence of the g_js, and continue to call the limit function g_0. Now let $\mathcal{T}_0$ be the graph of τ_0.

It follows that $\mathcal{T}_0$ is a totally real, two-dimensional manifold. And certainly g_0 vanishes on $\mathcal{T}_0$. Thus $g_0 \equiv 0$. Arguing as in the proof of Proposition 4, we conclude that f has admissible limit 0 at $\mathbb{1}$. □

3. Normal functions

We continue, for the moment, to work on the unit ball $B \subseteq \mathbb{C}^n$. Recall (see [3]) that a *normal* function has at least two equivalent definitions. Here we let $\hat{\mathbb{C}}$ denote the Riemann sphere.

Definition 6. Let $f : B \to \hat{\mathbb{C}}$ be holomorphic (here $\hat{\mathbb{C}}$ is the Riemann sphere). We say that f is *normal* if, whenever $\{\varphi_j\}$ are biholomorphic self-maps of B then $\{f \circ \varphi_j\}$ is a normal family.

In the paper [3] an important equivalent formulation was derived using ideas from invariant geometry.

Definition 7. A holomorphic function $f : B \to \hat{\mathbb{C}}$ is *normal* if the derivative ∇f is bounded from the Kobayashi metric on B (equivalently, the Poincaré–Bergman metric on B) to the spherical metric on $\hat{\mathbb{C}}$.

The equivalence of these two definitions is a sophisticated exercise with Marty's theorem. Now we have

Proposition 8. *Let $B \subseteq \mathbb{C}^2$ be the unit ball. Let $f : B \to \hat{\mathbb{C}}$ be holomorphic and normal. Let*

$$T = \{(s + i0, t + i0): \ s, t \in \mathbb{R},\ 0 < s < 1,\ 0 < |t| < \sqrt{2 - 2s}\}.$$

Suppose that $\rho : T \to \mathbb{R}^2$ is a C^2 function, with bounded first and second derivatives, such that (writing $\rho(s,t) = (\rho_1(s,t), \rho_2(s,t))$)

$$\mathcal{T} = \{(s + i\rho_1(s,t), t + i\rho_2(s,t)): \ (s,t) \in T\}$$

is a two-dimensional, totally real manifold in $B \subseteq \mathbb{C}^2$.

Let $\mathbb{1} = (1 + i0, 0 + i0)$. Suppose that

$$\lim_{\mathcal{T} \ni z \to \mathbb{1}} f(z) \equiv \lambda \in \mathbb{C}$$

exists. Then, for any $\alpha > 1$,

$$\lim_{\mathcal{A}_\alpha(\mathbb{1}) \ni z \to \mathbb{1}} f(z) = \lambda.$$

Proof. The key here is to let

$$\Phi : B \to B$$

be the automorphism

$$\Phi(z_1, z_2) = \left(\frac{z_1 + 1/4}{1 + (1/4)z_1}, \frac{\sqrt{1 - 1/16}z_2}{1 + (1/4)z_1}\right).$$

Then we define Ω_{j_0} as before but now we let

$$\Omega_j = \Phi^{j-j_0}(\Omega_{j_0}).$$

Here Φ^m is the mapping Φ composed with itself m times, $\ell > 0$.

Now the proof goes through just as that for Proposition 5. We merely must note that now we are examining the mappings $g_j \equiv f \circ \Phi^{-(j-j_0)}$. These are compositions of f with automorphisms. By the definition of "normal function," we may be sure that the g_j form a normal family. The proof is completed then as before. □

4. More general domains

It should be stressed that it is misleading, indeed essentially incorrect, to think of normal functions on an arbitrary domain in terms of automorphisms of the domain. For most domains in $\mathbb{C}$ or $\mathbb{C}^n$ have only the identity as an automorphism (see [8] for a discussion of this notion). One of the main motivations for the development in [3] of normal functions using the Kobayashi metric was to address this difficulty. Thus, if we wish to prove a Lindelöf principle on general domains, we certainly cannot use the ideas in Section 3.

Instead we examine the invariant Kobayashi metric. Let us begin by looking at a strongly pseudoconvex domain Ω with C^2 boundary. Let $P \in \partial\Omega$. By normalizing coordinates, we may assume as usual that $P = \mathbb{1} = (1, 0)$, that $\operatorname{Re} z_1$ is the real normal direction, and that $\operatorname{Im} z_1$ is the complex normal direction. Thus $z_2, \ldots, z_n$ are the complex tangential directions at P. For simplicity, we assume as above that the dimension $n = 2$ (the statements and proofs in higher dimensions are analogous). Following the paradigm in [7] or [6] (see also [12]), we may assume that there is an internally tangent ball at $P \in \partial\Omega$. There is no loss of generality to assume that this ball is the unit ball. Thus we may define regions Ω_j just as on the unit ball above. And the maps φ_j are defined as before.

Theorem 9. *Let $\Omega \subseteq \mathbb{C}^2$ be a strongly pseudoconvex domain with C^2 boundary. Let $f : \Omega \to \hat{\mathbb{C}}$ be holomorphic and normal. Let*

$$T = \left\{(s + i0, t + i0): s, t \in \mathbb{R}, \ 0 < s < 1, \ 0 < |t| < \sqrt{2 - 2s}\right\}.$$

Suppose that $\rho : T \to \mathbb{R}^2$ is a C^2 function, with bounded first and second derivatives, such that (writing $\rho(s,t) = (\rho_1(s,t), \rho_2(s,t))$)

$$\mathcal{T} = \left\{\big(s + i\rho_1(s,t), t + i\rho_2(s,t)\big): (s,t) \in T\right\}$$

is a two-dimensional, totally real manifold in $B \subseteq \mathbb{C}^2$.

Let $\mathbb{1} = (1 + i0, 0 + i0)$. Suppose that

$$\lim_{\mathcal{T} \ni z \to \mathbb{1}} f(z) \equiv \lambda \in \mathbb{C}$$

exists. Then, for any $\alpha > 1$,

$$\lim_{\mathcal{A}_\alpha(\mathbb{1}) \ni z \to \mathbb{1}} f(z) = \lambda.$$

Proof. It is propitious to consider a holomorphic mapping $\psi : D \to \Omega_{j_0}$, where $D \subseteq \mathbb{C}$ is the unit disc. If $\psi(0) = p \in \Omega_{j_0}$, then we may take ψ to be an extremal function for the Kobayashi metric at the point p. Now look at $\mu_j : D \to \hat{\mathbb{C}}$ given by $f \circ \varphi_j^{-1} \circ \psi$.

Then we calculate that

$$\left|\mu_j'(0)\right| \leqslant \left|\nabla f\left(\varphi_j^{-1}(p)\right)\left[\left(\varphi_j^{-1} \circ \psi\right)'(0)\right]\right|. \tag{$*$}$$

Of course the second expression in brackets on the right is nothing other than the reciprocal of the Kobayashi metric for Ω_j at $\varphi_j^{-1}(p)$. The first expression, as we know from the definition of "normal function," is bounded from the Kobayashi metric on Ω (which is smaller than the Kobayashi metric on Ω_j) to the spherical metric on $\hat{\mathbb{C}}$. In sum, the expression $(*)$ is bounded on compact subsets of D. And this bound is certainly independent of j, and also independent of the choice of p—as p ranges over a compact subset K of Ω_{j_0}.

We may obtain a similar estimate (just by composing with a Möbius transformation) for μ_j' at any point of a compact subset L of D. Of course the estimate will depend on L.

So we may extract a normally convergent subsequence of μ_j'. Call the limit function μ_0. Arguing now as in the proof of Proposition 5, we see that there is a corresponding subsubsequence τ_{j_k} converging to τ_0 and a limiting totally real, two-dimensional manifold $\mathcal{T}_0$ in Ω_0 which is the graph of τ_0. Thus we find that μ_{j_k}' converges on $\mathcal{T}_0$. Putting together the convergence of the derivatives together with the convergence of the functions on $\mathcal{T}_0$, we see that the functions themselves converge uniformly on compact subsets of Ω_{j_0}. By the usual logic, we find that f has admissible limit λ at P. $\square$

Note that, with some additional effort, Theorem 9 may be extended to domains of finite type in $\mathbb{C}^2$ (see [5,9]).

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